

2005年度 微積分学II 演習問題(4)

1. 次の関数 $f(x, y)$, $\varphi(t)$, $\psi(t)$ について, 合成関数 $z(t) = f(\varphi(t), \psi(t))$ の微分 $z'(t)$ を求めよ.

(1) $f(x, y) = x^2 + xy + y^3$, $\varphi(t) = t^2 + 1$, $\psi(t) = 2t$

(2) $f(x, y) = xy^4 + x^2y + 3y^2$, $\varphi(t) = t^3$, $\psi(t) = t^5$

(3) $f(x, y) = \sin(2x - y)$, $\varphi(t) = 3t^2$, $\psi(t) = 5t^2$

(4) $f(x, y) = xy$, $\varphi(t) = e^t$, $\psi(t) = e^{-t}$

(5) $f(x, y) = x^2 + y^2$, $\varphi(t) = \sin t$, $\psi(t) = \cos t$

(6) $f(x, y) = xy^2 + x^3$, $\varphi(t) = \sin 2t$, $\psi(t) = \cos 3t$

2. $z = f(x, y)$ が C^2 級るとき, 次の関数 $z(t)$ に対し, $z'(t)$ と $z''(t)$ を求めよ.

(1) $z(t) = f(2t, 3t)$

(2) $z(t) = f(t^2, t^3)$

(3) $z(t) = f(e^{2t}, e^{3t})$

(4) $z(t) = f(\sin t, \cos t)$

3. 次の関数 $z = f(x, y)$, $\varphi(u, v)$, $\psi(u, v)$ について, 合成関数 $z(u, v) = f(\varphi(u, v), \psi(u, v))$ の偏微分 z_u, z_v を求めよ.

(1) $f(x, y) = x + 2y$, $\varphi(u, v) = u + v$, $\psi(u, v) = uv$

(2) $f(x, y) = xy$, $\varphi(u, v) = u + v$, $\psi(u, v) = u - v$

(3) $f(x, y) = x^2 + y^3$, $\varphi(u, v) = u \cos v$, $\psi(u, v) = u \sin v$

(4) $f(x, y) = x^2 + xy + y^2$, $\varphi(u, v) = u \cos v$, $\psi(u, v) = u \sin v$